MATHEMATICS INSTRUCTION AND MATHEMATICAL VOCABULARY: ENGAGING A NEURODIVERGENT STUDENT

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This study explores volitional and affective responses to instructional activities aimed at developing geometric and spatial vocabulary of a neurodivergent student. Using teaching experiment methodology across 15 instructional sessions, we observed how the student responded to games, direct instruction, and vocabulary support embedded in spatial reasoning activities. A narrative microgenetic analysis explored how these activities were associated with evidence of engagement and confidence in learning. We describe how embedding vocabulary in the student's own mathematical activity was supportive of developing vocabulary, engagement, and confidence, while games and direct vocabulary instruction were not.

Keywords: Students with Disabilities; Instructional Activities and Practices; Affect, Emotion, Beliefs, and Attitudes; Geometry and Spatial Reasoning

As mathematics educators we aim to *engage all learners*, believing that mathematics learning should be a joyful experience for all students. Too often, students with difficulty in mathematics do not experience instructional activities in this way. The PME-NA 2023 conference theme reflects our desire to facilitate students' engagement and interest which in turn will support math learning.

Our research project aimed to encourage positive beliefs and attitudes around mathematics learning in Eva, a neurodivergent student who has had few opportunities for success in the school setting. We conducted a teaching experiment to find instructional activities that were supportive of Eva's learning. We used a narrative microgenetic analysis to characterize the interaction between the instructional activities and Eva's volitional and affective responses.

Theoretical Perspective

Advocating for rehumanizing the mathematics education of students with disabilities, Lambert and colleagues (2018) emphasizes the need to disrupt persistent deficit perspectives found in research. They call for the use of multiple theoretical frameworks to further our understanding of the relationship between disability and mathematics learning. We aimed to participate in rehumanizing the mathematics education of students with disabilities in this research that explores instruction grounded in different theoretical perspectives. We recognize Eva as an individual whose cognitive and affective being is not in harmony with typical educational settings, and we take the position that deficits are present in our instruction. Therefore, we conducted this research with Eva, a nine-year-old student with considerable difficulty in mathematics which may stem from instructional environments which are not responsive to her language processing, executive functions, and anxiety. We investigated how Eva responded to instruction which reflects different theoretical perspectives on teaching with the goal of understanding how to make mathematics learning a joyful experience.

Individual Differences

If we are to engage all learners, we need to identify instructional approaches that are responsive to cognitive diversity and volitional or affective processes which are associated with mathematics outcomes. Cognitive differences related to mathematics achievement vary across domain-specific, domain-general, and executive functioning processes (Ashkenazi et al, 2013; Clements et al.,

2016). Additionally, volitional and affective processes such as motivation, inhibitory control, and anxiety can affect how a student experiences the mathematics learning environment (Nelson et al., 2022). Thus, a variety of individual differences in processes may need to be considered when tailoring instructional support for an individual student.

Eva has difficulty with language processing, executive functions, and anxiety. Language processing, the most common of neurodevelopmental disorders (Koerte et al., 2016), may include difficulty with auditory processing, receptive and expressive vocabulary knowledge, and semantic processing (Hämäläinen et al., 2013; Roesch, 2019). Further, extensive research has identified associations between executive functions and anxiety with success in mathematics learning (Clements et al., 2016; Nelson et al., 2022; Pérez-Fuentes et al., 2020). Thus, several processes may impact Eva's learning of geometric and spatial vocabulary.

Spatial Reasoning and Language Processing

Research on spatial reasoning in students with atypical language processing is limited and inconclusive. Meta-analyses indicate that individuals with dyslexia, a language processing disorder, may perform better, the same, or worse than peers depending upon the nature of the task; a consistent finding is greater variability in their performances as compared to peers (Chamberlain et al., 2018; Gilger et al., 2016). A recent study found that geometric processing was associated with short-term memory skills, while arithmetic and measurement skills were associated with receptive vocabulary (Yang, 2023). We were not able to locate research specifically reporting on language processing and geometric or spatial orientation vocabulary. More research is needed to determine if there are patterns which offer insights for instruction.

Behaviorist Learning Theory, Constructivism, and Learning Environments

Games have long been used to support mathematics learning (Bright et al., 1985; Pan et al., 2022; Siegler, 2009). Pan and colleagues (2022) recently conducted a systematic review of the role of games in mathematics education. They found that behaviorist learning theories were present in nearly 60% of the games studied, followed by experiential learning, and finally constructivist theories. Overall, the research has shown small, positive effects on achievement and self-efficacy, but there are a number of studies with contradictory results. Further, the degree of generalizability is murky across instructional context and student populations (Pan et al., 2022). This review suggests it is not clear what underlying instructional theory and what game formats are likely to have the most benefit for Eva.

For many years there has been disagreement between the fields of special education and mathematics education about approaches to instruction (Woodward, 2004; Munter et al., 2015). Many researchers, particularly in special education, contend vocabulary instruction should be explicit, providing precise, student-friendly definitions supported with examples, non-examples, and representations (Fuchs et al., 2021; Munter et al., 2015). Further, vocabulary should be used repeatedly throughout a lesson or lessons to reinforce students' understanding and corrective or affirmative feedback provided as appropriate (Fuchs et al., 2021; Hattie & Timperley, 2007). These approaches are linked with behaviorist learning theory (Woodward, 2004). Aims are to control cognitive load and ensure successful performance (Grigorenko et al., 2020: Fuchs et al., 2021; Kalyuga, 2009).

In contrast, many researchers in mathematics education and some in special education contend that vocabulary and definitions can be developed with constructivist instructional processes (Ball & Bass, 2000; Lampert, 1990; Munter et al., 2015). Students participate in creating definitions, building on their own experiences or current understandings. The teacher's role is to ensure that students construct mathematically sound definitions which emerge from their ideas and activity

(Freudenthal, 1968; Munter et al., 2015). One argument for this approach is that instruction which starts from student thinking is consistent with a strengths-based approach believed to support positive volitional and affective outcomes (e.g., motivation and beliefs about self) and academic outcomes (De Corte et al., 2000; Rappolt-Schlichtmann et al., 2018). There is a growing number of studies that intentionally test the effects of instruction grounded in constructivism with neurodiverse students or students with or at-risk for difficulty in mathematics (e.g., Baroody et al., 2012; Gervasoni et al., 2021; Hunt et al., 2020). However, research is needed on the effects of these instructional approaches for vocabulary development.

Research Purpose

We aimed to better understand the features of instructional activities which support Eva's spatial reasoning and geometry understandings as well as positive volitional and affective responses. Specifically, we asked: What instructional activities help Eva develop geometry and spatial orientation vocabulary in ways that are also supportive of her engagement and confidence? We analyzed Eva's responses to vocabulary-rich games, direct instruction in vocabulary, and embedding vocabulary in Eva's mathematical activity.

Method

This analysis is part of a larger project in which we investigated Eva's mathematics learning through a teaching experiment centered on established learning trajectories (Crawford & Kernin, 2022). The teaching experiment was followed by a narrative microgenetic analysis. Teaching experiments are forms of design research which systematically investigate teaching and learning in naturalistic settings (Confrey & Lachance, 2000; Steffe et al., 2000). After the teaching experiment, the narrative microgenetic analysis focused on identifying conditions that might promote learning (Lavelli et al., 2005; Siegler, 2006).

Participant Information

Eva, a multi-racial girl, was nine years old and in grade 3 at the time of this study. When she was seven years old, Eva was given a neuropsychological evaluation which identified her as having attention deficit with unspecified impulse-control and conduct disorder, speech-sound disorder, specific language disorder with impairments in written language and mathematics, and generalized anxiety. Eva has received special education services in a pull-out program ("resource room") at a public school and a public charter school in a small city in the western United States. Upon meeting with Eva's mother, we discussed ways to provide her with additional academic support and be mindful of her anxiety. We agreed Eva's mother would be present for all teaching experiment sessions. We gained informed parent consent and student assent following guidelines established by the university's Institutional Review Board.

We recognize our roles as participants in this research. One of us, Angie, conducted the teaching experiments in relationship with Eva and her mother. This relationship involved establishing trust and sharing authority through negotiating goals, activities, what was to be attended to and what was not. As researchers we, Angie and Aysia, continue this relationship with Eva as we interpret her activity and make inferences about her engagement and confidence. We take the position of bracketing ourselves into this narrative, rather than attempting to position ourselves outside (Connelly & Clandinin, 2006).

Teaching Experiment Procedure

One author, Angie, conducted the teaching experiment sessions. All sessions took place at Eva's home, and Eva's mother was present to support Eva's well-being. The sessions took place in 2021 amidst the COVID-19 pandemic, and meeting dates were flexible to allow for family needs and local quarantining requirements. Due to the pandemic and to limit factors that might contribute

to Eva's anxiety, it was not possible to have an additional researcher present during the sessions. There were 15 teaching experiment sessions, each ranging from 30-45 minutes.

Tasks described in this report come from spatial reasoning learning trajectories developed with children from birth to grade 3, specifically the spatial orientation and 2D shapes and shape composition trajectories (Clements & Sarama, 2021). The spatial orientation trajectory involves relationships that position objects in space, including vocabulary such as "on," "under," "beside," etc. The 2D shapes trajectory involves learning to name, describe, and classify two-dimensional shapes using properties of the shapes. The 2D shape composition trajectory involves putting together two-dimensional shapes to make other larger, composite shapes.

Angie used a planning protocol to document instructional decisions and reflections on teaching experiment sessions. The protocol included prompts for observations, outcomes, adjustments made, and rationale for adjustments. Then plans for the next session were developed based on the reflections. The protocol and plans were shared with two colleagues. One colleague, with expertise in early mathematics instruction for students with learning disabilities, advised on the tasks and supports. A second colleague, with experience with children with anxiety, advised on the plans in light of Eva's affective responses.

Data Sources

During each session, Angie asked Eva's permission to video record the activity and only recorded when Eva stated she was comfortable with it. We recorded nine of the 15 sessions. Angie took field notes during each session. Therefore, data sources were the planning protocols previously described, field notes from each session, photographs of student work, videos and transcriptions, and a post-hoc observation protocol.

The other author, Aysia, completed post-hoc observations using a protocol as a way to triangulate observations and interpretations of Eva's responses to activity. This protocol recorded: the task description, student behaviors (strategies, demonstrations, comments), teacher behaviors (explanations, providing time to work without interruption, additional supports), and any other observations. Aysia completed the protocol for each video.

Data Analysis

We used a microgenetic narrative analysis which can account for the multidimensional nature of learning (Lavelli et al., 2005). This approach comprises five stages (Lavelli et al., 2005). The first involves watching videos to identify a list of potential "frames," the specific lenses which might serve as focal points for analysis. Tasks, language use, vocabulary, and affective responses were among the potential frames we identified. In the second stage, we constructed descriptive narratives of each session of the teaching experiment to document the sequence of events. To ensure accurate and comprehensive narratives, we each wrote a draft narrative for half of the sessions and then reviewed and revised the narratives written by the other. We recorded thoughts related to frames, interpretations, questions, or reflections in a parallel set of memos. In the third stage, we used the descriptive narratives to discuss possible combinations of frames that captured interactions influencing learning. This report presents the vocabulary and Eva's affective responses frames. The fourth stage involved re-reading the descriptive narratives to develop the frames into a meaningful and plausible "plot." We sought evidence to confirm or refute this plot through an in-depth analysis of the data sources with particular attention to overt behaviors and strategies. In the fifth and final stage of analysis, we used the plot and evidence to create a narrative that synthesizes our views on Eva's experience.

Findings

This narrative microgenetic analysis tells the story of our efforts to identify features of instructional activity which support Eva's geometry and spatial orientation vocabulary as well as positive volitional and affective responses. Specifically, we ask: What instructional activities help Eva develop geometry and spatial orientation vocabulary in ways that are also supportive of her engagement and confidence? We describe Eva's responses to vocabulary-rich games, direct vocabulary instruction, and vocabulary support embedded in spatial reasoning activities.

Vocabulary-rich Games

Angie planned games which hinged on geometric or spatial vocabulary in activity. One example is the game "I Spy," used with the spatial orientation learning trajectory. In this version of the game one person states they are thinking of an object that is located in relation to other objects, identified by spatial vocabulary such as besides, above, between, etc. The other person needs to determine what the secret object is.

We saw considerable evidence of Eva's positive engagement with the "I Spy" game. Eva asked to play it during more than one session. When Angie and Eva played in Eva's backyard, Eva enthusiastically ran about to search for the object. When played seated at a table, Eva often rocked back in her chair and appeared relaxed. Eva eagerly took turns being the person who had a secret object in mind and the person who sought the object.

However, there was little evidence that her vocabulary was developing. She was able to locate objects when the clue contained one, familiar spatial term such as "on" or "under." When other terms were substituted (e.g., "below") or added (i.e., multi-part clues such as "under" and "beside"), Eva was not able to locate the object. In these instances, Eva appeared to search for the secret object based on adjectives rather than use the spatial orientation clues. For example, when Angie gave her the clue, "I spy something blue beside a tree," Eva looked for anything blue, whether or not it was near a tree–even when Angie repeated the phrase "beside a tree" several times or used "beside" with the next object. When it was Eva's turn to select a secret object, she would describe the appearance or function of the object rather than use spatial orientation terms.

Angie tried another game that pressed Eva to use vocabulary more intentionally. Angie used pattern blocks and connecting cubes to make a simple design which was hidden from Eva's view by a screen. Angie asked Eva to make a copy of the design based on the verbal cues provided. Eva found this activity frustrating. For the first two designs, Eva selected and placed pattern blocks and cubes based on known attributes (e.g., red, triangle) without orienting the object as described (e.g, "next to the square") and thought she was correct. On the second design, she reached out and grabbed the screen so that she could copy the design. The following transcript picks up after Eva has been told by her mom and Angie that she cannot grab the screen:

Eva: Why does everybody have to be super, super intense?
Angie: Because you don't learn as much when you look.
Mom: Want to try it again to see if you can listen and get it? Or is it too hard for you?
Eva: Too hard
Angie: Okay then let's do it this way. [adjusts by giving Eva the three brown cubes which are needed] Okay. I would like you to put one of these in front of you….One behind it, and one in front of it….One in front of you…and then put one behind it and one in front of it…..And now, can you take one more?…. Do you have red or brown?…One more brown and put it on top of the one in the middle. You have one in front and one behind and one on top of the one in the middle.

Eva: Everything is square. [Eva’s design does not match Angie’s]
Angie: Okay. Put one in front of you.
Eva: I know it's a square. I know.
Angie: Now put one behind it.
Eva: It is behind.
Angie: And one in front of it.
Mom: In front.
Angie: And one on top.
Eva: Too hard
Mom: Watch me. You give me direction so you can kind of get an idea of what in front and on top means. Okay. Just watch, please stop getting frustrated.
Eva: I’m watching [holds up the plastic lid of a box in front of her face]
Mom: Can you watch please? Okay what are we doing?
[Angie and Mom go through same sequence placing brown cubes]
Mom: Did you understand it?
Eva: No.
Mom: Do you want to try?
Eva: No. Too hard.
Angie: Do you want to tell us [cut off by Eva before finishing the question]
Eva: Too hard. Too hard. Too hard.
Angie: Okay, that's alright. It's too hard, too hard, too hard. That’s honest.
Eva: At least I tried.

This transcript shows how Eva struggled to process the spatial orientation vocabulary. Unlike the “I Spy” game, Eva did not show enjoyment or confidence with this activity and asked several times to disengage. She seemed to find the mystery of the games enjoyable unless she had difficulty processing the multiple clues or when pressed to rely only on spatial orientation clues. When that difficulty arose, she either resorted to random searching or wanted to abandon the activity. Despite some evidence of engagement, there was little evidence that she was developing understanding of the vocabulary or confidence in her knowledge.

Direct Vocabulary Instruction

Angie used direct instruction to link shape names with shape attributes, aiming to deepen Eva’s understanding of the geometric vocabulary. During the first session, Eva was able to identify circles, triangles, and squares. So, in a subsequent lesson, Angie explained the three features that make a circle and focused on terminology such as “curve,” (i.e., a curved line, no gaps or breaks in the line, curve stays the same all the way around). Angie gestured with an example and non-example and repeated the three features. Eva played with a sensory toy intended to help focus attention but did not show outward signs of listening. Angie asked Eva to repeat back one of the features. Eva said she did not remember. Angie then asked Eva to find something curved. Eva pointed to a square-shaped cushion with rounded edges, straight sides, and flat surfaces. Angie pointed out parts of the cushion that were straight and curved. Eva started playing with and chasing her pet dog who had been sitting at her feet.

In the next lesson, Angie asked Eva if she remembered the three features of a circle. Eva’s response was, “No, but I bet you’ll tell me.” After naming the features, Angie asked Eva to identify which shape among several drawn on a card was not a circle and explain why it was not. Eva

pointed to the correct shape but her explanation was, “It doesn’t go right.” Eva then began to tell a story about seeing circles on the playground.

During another session, Angie wanted to explicitly discuss the difference between curved and straight lines. The following transcript comes from this session:

Angie: Okay. Are you ready?
Eva: Now I’m paying attention
Angie: Okay, number 1 [pointing to numbered cards with two-dimensional shapes]. You see the one that has a number 1 is not a circle, because it…? What are its sides?
Eva: Triangle
Angie: It has straight sides. So, can you repeat back for me why number 1 is not a circle?
Eva: Because it's a triangle,
Angie: and it has ….?
Eva: Four sides

It is not clear if the error at the end, when Eva says the triangle has four sides, is simply an error due to lack of attention, misspeaking or mis-counting, or something else. We note, though, that the error occurred when Eva was pressed to respond with reference to an attribute. It is also evident in these descriptions of direct instruction on attributes of circles that Eva’s engagement was limited. Distractions were prevalent and Eva’s effort to answer questions was limited to pointing or giving simple, short answers. Looking over her responses in aggregate, responses such as “I bet you’ll tell me,” pointing to a cushion, and “because it’s a triangle,” we considered the possibility that her engagement was perfunctory.

Vocabulary Embedded in Activity

As Eva worked on activities in the 2D shape composition tasks, Angie gave feedback that incorporated shape names or spatial transformation vocabulary. In the early sessions, Eva relied on trial-and-error to find the correct shape to fill in a larger shape puzzle. She gradually began to select pattern blocks with intention based on appearance and turn them to orient them correctly. Angie encouraged this with positive feedback, “Wow! What great problem solving! I saw you *turn* the block to find out if it would fit another way.” In the next session, Eva was using trial-and-error again to fill in an outline of a dog. Eva almost gave up when she could not place a shape to fill in the outline that indicated a trapezoid. Angie pulled out three pattern blocks and said it was one of these and that she could find it if she used “turning” again. Eva selected the trapezoid and turned it until it was oriented correctly. Angie then said, “Yes! You saw that it was a trapezoid that would fit if you turned it.” In the next session, Eva intentionally selected and placed trapezoids to create a hexagon in the center of a picture. Later, after Angie and Eva’s mom had used the name of the shape on several occasions, Eva had difficulty providing the full term but could approximate it (e.g., “tra-me-zo”).

Also, we noted that Eva’s confidence grew, and she was able to solve increasingly complex 2D shape composition tasks. For example, in the eleventh session Eva demonstrated engagement and confidence in her ability to solve tasks which involved composing shapes without interior lines provided as cues:

Angie: Can you choose a couple of those to do? Thank you.
Eva: Oooh! [Looks through puzzles and chooses one without interior lines]
Angie: You want to do that one?
Eva: Hard.

Angie: I also have some others in here that are even more hard but I thought [Eva starts talking before Angie finishes statement]

Eva: This looks like a camel [begins looking for pattern blocks to fill in the shape]

Angie: A camel? It kind of does. Yes, it's got a hump on its back.

Eva: Yep. This is hard, this is hard.

Angie: What do you think seems hard about that one?

Eva: Everything.

Angie: Pardon me? What?

Eva: Everything.

Angie: What do you mean "everything"? It seems like you knew what to do right away.

Eva: No I mean like it's going to be hard for friends, and mom or dad, I mean that.

We see evidence of Eva's engagement in the interest she showed in solving the puzzle. Further, there is clear evidence in her confidence in solving this puzzle by stating that everything seemed hard about it, but it would be hard for others like her friends or parents.

Discussion

To identify instructional activities that were enjoyable for Eva and helped her to develop geometric and spatial orientation vocabulary, we tried a variety of formats: vocabulary-rich games, deliberate vocabulary instruction, and vocabulary development embedded in other activity. We found that only vocabulary embedded in Eva's mathematical activity seemed to have positive effects on her understanding of terms, engagement, and confidence.

Eva was engaged in some of the vocabulary-rich games but not others. Also, we found that repeatedly using a particular term across several iterations of the activity did not seem to help her apply or reason with that term. This may be explained by ways in which the game increased cognitive load through requiring her ability to process verbal cues without other supports (Kalyuga, 2009). Our finding suggests one possible reason why studies of games have had contradictory results which limit generalizability (Pan, 2022).

We also saw little success with direct vocabulary instruction. Eva showed little sustained engagement in these lessons and responded in what seemed perfunctory ways. This is despite recommendations for this approach on the basis of research evidence from experimental and quasi-experimental studies for its efficacy for students with difficulty in mathematics (Fuchs et al., 2021). We believe this discrepancy highlights the fact that quantitative research studies report aggregated results. As such, the conclusions may not apply or have limited efficacy when used on an individual, case-by-case basis.

We found the most success in developing vocabulary, confidence, and engagement when the vocabulary support mathematized Eva's activity (Freudenthal, 1968). As a result, Eva demonstrated understanding of the terms, sometimes used the terms herself, and remained engaged with the activities during each session. Over time, she expressed confidence in her ability to complete these tasks. The vocabulary development with this instructional approach may be attributed to the combination of presentation without increasing cognitive load and the positive, specific feedback she received (Kalyuga, 2009; Hattie & Timperley, 2007).

We engaged in this research to learn how we might help Eva to develop mathematical vocabulary in ways that supported her engagement and confidence. Our findings suggest that for Eva an instructional approach that engages, builds confidence, and hopefully leads to a joyful experience, is when her activity is mathematized and, as a result, valued.

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